Remarks on Strongly Deformed Nuclei

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Nuclides in various regions of the Nuclide Chart have shapes that are characterized by sizeable stable, axially symmetric, quadrupole deformations. One of the prominent features in the level schemes of these nuclides is the systematic occurrence of well-developed rotational bands. A rotational band is a sequence of levels associated with a given intrinsic state, or "band head", which arise from the addition of rotational energy to the nuclide in that particular state. This rotation takes place about an axis perpendicular to the nuclear symmetry axis. Any intrinsic state (one-quasiparticle, multi-quasiparticle, vibrational, etc.) may constitute a band head.

In the coupling schemes appropriate to these nuclides, the intrinsic (i.e., nonrotational) and the rotational motion can, to a first approximation, be separated, with the result that the matrix elements of various operators can be expressed as products of nonrotational and rotational terms. This leads to a great simplification in the calculation of a number of properties of these levels, since the rotational portion can frequently be evaluated using relatively simple "geometric" considerations.

In the simplest picture, the band members have energies that vary with spin as J(J+1), a common K value and intraband gamma transitions that are characterized by large transition quadrupole moments. To a first approximation, the band members have a common intrinsic state. However, residual interactions may lead to some admixtures of different intrinsic states.

(In instances where strong octupole-correlation effects - e.g., parity-doublet bands - are present, strong E1 transitions, which connect adjacent-spin band members, are observed. Also, the recent observation, in the high-energy excitation spectrum of certain nuclides with near spherical symmetry, of a "band structure" whose members are connected by M1 transitions and result from different orientations of the angular-momentum vectors of the residual protons and neutrons may represent a situation which is qualitatively different from that considered here.)

Note: In the following, we use the terms “single-particle” state and “quasiparticle” state more-or-less interchangeably. Also, the reference list is not intended to be complete. It simply includes references that illustrate the points under discussion. It is heavily weighted toward works with which the author has been personally involved. The reader can doubtless come up with many more. A number of excellent review articles [e.g.,1-4] exist, which provide considerably more depth than is given in this presentation and to which the interested reader is referred for additional information.

I. **Wave Functions and Energetics**

In the presence of nuclear deformation, the degeneracies inherent in the various spherical shell-model states are broken and their quantum numbers (nlj) are no longer good ones. A typical dependence of the resulting "single-particle" states on the nuclear deformation is shown on the so-called "Nilsson diagrams" in Figs.1a and 1b, taken from [5].

Here, a good quantum number is K, the projection of the particle's total angular momentum on the nuclear symmetry axis. The energy levels are labeled by the so-called asymptotic quantum numbers [N n_z Λ]. K= Λ + Σ, where Λ and Σ (= ± ½) are the projection of the particle's orbital angular momentum and intrinsic spin, respectively, on the nuclear symmetry axis. Each such Nilsson state (or "orbital") consists of a mixture of the various spherical shell-model states, with amplitudes C_{jl}, and can accommodate only two particles, having projections K and -K.

The asymptotic quantum numbers are quite useful in assessing the properties and interactions of the particles occupying the levels (or Nilsson "orbitals"). This is one of the features of the strongly deformed nuclei that make their study so interesting for the average experimentalist and so helpful for the nuclear-data evaluator. Useful selection rules involving the asymptotic quantum numbers, as summarized in [5], are shown in Fig. 2.

If the couplings among the states are not too strong, the energies of the band members can be expressed by the relatively simple expression (see, e.g.[1] and references therein):

$$E(J,K) - E_K = AX + BX^2 + CX^3 + ... + (-1)^{J+K} \prod_{i=1-K}^{K} (J+i)\{A_{2K} + B_{2K}X + ...\} \quad (1)$$

where $X = J(J+1)-K^2$.

The last term in Eq.(1) describes the "staggering" that arises from the coupling with other bands. For the present discussion, we simply call attention to this term for K=1/2 bands. In these cases, there occurs the so-called decoupling parameter, *a*, which is characteristic for each K=1/2 band. It is given by the ratio A_1/A in Eq(1).

In view of the close connection between level energy and level spin within a rotational band, in making Jπ assignments the evaluator may take into account rotational-band considerations as well as the usual arguments regarding possible Jπ values. This frequently makes it possible to make Jπ assignments with great confidence (strong arguments) from data which would not otherwise (e.g., in regions of the Nuclide Chart where rotational bands are not found) be conclusive.

Considerations involving the use of Eq.(1) in nuclear data evaluation are presented in more detail in the article on deformed nuclei in Procedures Manual for Evaluators [6] and are not discussed further here.

II. <u>**Configuration Mixing**</u>

The Nilsson states illustrated in Fig. 1 constitute a good approximation to the actual intrinsic (nonrotational) states
occurring in real nuclei. However, residual interactions introduce admixtures into the wave functions of the nuclear states that can significantly affect their properties. We now briefly summarize some of the more important of these.

A. *Rotation-particle (Coriolis) Coupling*

The nuclear rotation induces a mixing of Nilsson states. This so-called Coriolis coupling mixes states having the same $J\pi$ values but (to first order) K values that differ by one unit. This interaction repels the two states involved, with the result that, Coriolis-mixed rotational bands appear compressed or expanded, depending on whether they were initially below or above, respectively, each other. For a given Nilsson state, this interaction is strongest among adjacent-K states originating from the same spherical shell-model state (see Fig. 1). It increases with increasing spin (J) and is stronger the higher the j-value of the originating spherical shell-model state. Thus, in the odd-mass nuclei, Coriolis effects are strongest among the $i_{13/2}$ neutron states and the $h_{11/2}$ proton states in the rare-earth region and the $j_{15/2}$ neutron states in the actinide region.

Coriolis effects are also especially prominent in the octupole-vibrational states in the even-even nuclides of the rare-earth elements, as discussed in detail in [7,8]. In fact, observation of significant Coriolis-related effects can provide very helpful information regarding the configurations of the nuclear levels involved.

A more detailed discussion of the features of this interaction, including the form of the relevant matrix elements, the dependence on the asymptotic quantum numbers and how it can significantly affect various nuclear phenomena, is given in the relevant section of [6].

Note: Although the admixtures induced by Coriolis coupling may be rather small, when the admixture carries with it a large matrix element for a given process (e.g., an *au* β transition, a "rotational" E2 transition or a favored β transition), then the admixture may have a dominant effect on that process.

B. *Quasiparticle-phonon Coupling*

In the even-even nuclei, a prominent systematic feature of the level scheme is the occurrence, below the pairing gap, of collective excitations having $K\pi=2+$, called Gamma Vibrations. In the odd-mass nuclei, there can be two gamma-vibrational states associated with each one-quasiparticle state - the base state - (of K-value K_0), having K-values $|K_0-2|$ and K_0+2. Of these two, the one having the smaller K-value lies below that having the larger K-value. These excitations are frequently found to be strongly mixed with one-quasiparticle states. As discussed, for example in [1], this mixing can be large when the one-quasiparticle state occurs near the gamma vibration and where they are connected to the base state by a large E2 ($Y_{2,\pm2}$) matrix element. As indicated in Fig.2, their asymptotic quantum numbers are related as follows: $\Delta N=\Delta n_z=0$, $\Delta\Lambda=\Delta K=\pm2$.

For pure gamma-vibrational states with K=1/2, the decoupling parameter is expected to be zero. When a one-quasiparticle state is mixed into a gamma vibration, the decoupling parameter is no longer zero, but is generally smaller than that expected for the pure one-quasiparticle state.

C. *Mixing of two-neutron and two proton excitations*

In even-even deformed nuclei, two-quasiparticle excitations of both two-proton and two-neutron character occur near and above the pairing gap. These excitations are sometimes found to be strongly (30% or more) mixed. Perhaps the best known and documented of these cases is that of the two $K\pi=8-$ bands in ^{178}Hf, as discussed, for example, in [9].

III. **Electromagnetic Transition Probabilities**

The following expressions for E1, M1 and E2 transition probabilities are relevant to this discussion.(For an excellent and more comprehensive treatment, see the section on Transition Probabilities in [6].) We do not treat the effect of internal conversion here, since it should be done as is customary in such circumstances. In terms of the disintegration constant, λ, for gamma emission, we have

$$\lambda(E1)\downarrow = 1.59 \times 108\ (E\gamma)^3\ B(E1)\ \mathrm{sec}^{-1}, \qquad (2)$$

$$\lambda(M1)\downarrow = 1.76 \times 104\ (E\gamma)^3\ B(M1)\ \mathrm{sec}^{-1}, \qquad (3)\ \text{and}$$

$$\lambda(E2)\downarrow = 0.01224\ (E\gamma)^5\ B(E2)\ \mathrm{sec}^{-1} \qquad (4).$$

Here, $E\gamma$ is the gamma energy in keV, B(E1) and B(E2) are in e^2b and e^2b^2, respectively, and B(M1) is in $(\mu_N)^2$.

For gamma transitions of multipolarity L from a state of spin J in a band to two members of another (or the same) band having spins J' and J", the Alaga rules [10] state that the ratio of the reduced transition probabilities is given by the ratio of the squares of the relevant Clebsch-Gordan coefficients, viz.

$$\frac{B(L; J \to J'')}{B(L; J \to J')} = \frac{(C\begin{smallmatrix} J & L & J' \\ K & \Delta K & K' \end{smallmatrix})^2}{(C\begin{smallmatrix} J & L & J' \\ K & \Delta K & K' \end{smallmatrix})^2}$$

A. *E1 Transitions*

For E1 transitions between "single-particle" states, the Alaga rules do not generally give good agreement with experiment. This arises, at least in part, because all such transitions are *asymptotically hindered*, as inspection of the selection rules in Fig. 2 indicate.
The orbitals that are connected by *unhindered* E1's lie in the adjacent major shells and thus are not found among the low-lying states of the deformed nuclei.

Nonetheless, it is found that, in some cases, the reduced E1 transition probabilities are well described by the Alaga-rule predictions. Perhaps the best studied of these cases involve the octupole-vibrational bands in the even-even nuclei [7,8]. The good agreement here, which involves transitions with both ΔK=0 and ΔK=1, suggests that the Alaga rules are obeyed by collective E1's. Thus, if in an analysis of E1 branching, one finds that the Alaga-rule predictions are obeyed, it is reason for suspecting that the E1's are collective and that, for example, octupole-related effects are involved.

B. *E2 Transitions*

These play a prominent role in the study of the deformed nuclei, owing to the fact that intraband transitions are governed by the
large intrinsic quadrupole moment, Q_0, of the nucleus. The B(E2) values in these cases are given by the expression

$$B(E2; JK \to J'K) = \frac{5}{16\pi} e^2 Q_0^2 (C\begin{smallmatrix} J & 2 & J' \\ K & 0 & K \end{smallmatrix})^2$$, with Q0 in barns.

If the intrinsic quadrupole moment is known or can be estimated, as often the case, then the absolute B(E2) value for an intraband E2 transition can be inferred. From this, absolute transition probabilities (and hence the level lifetime) can be deduced with reasonable confidence if the gamma branching is known. Further, for the deexcitation of a band member to two lower-lying members of the same band, the Alaga rules can be used to deduce the E2 component in the cascade (ΔJ=1) transition, relative to the crossover (ΔJ=2) one, from which the M1 component in the cascade transition can be deduced. Using Eq.(3) and the expression

$$B(M1;JK \rightarrow J'K) = \frac{1}{4}\frac{3}{4\pi}(2K)^2\left(C^{J\ \ l\ \ J'}_{K\ \ 0\ \ K}\right)^2(g_K - g_R)^2$$

for B(M1) values within a rotational band, one can extract values for the g-factors of the cascade transition or at least of the ratio $(g_K - g_R)/Q_0$. These quantities should be reasonably constant within a band and can yield important information about the configuration of the band involved, as has been demonstrated, for example, in [8,9].

IV. **Beta Transitions**

Alaga-rule relations have been formulated for relative β branching from a given state to members of daughter-nucleus band, but in practice these are not very useful.

The important aspect of β decay that is so helpful in the analysis of data for the deformed nuclei is the occurrence of so-called *allowed-unhindered* (*au*) β transitions. These are transitions for which the asymptotic quantum numbers of the orbitals involved in the transition do not change (cf. Fig. 2). Such transitions have small *log ft* values. Among the deformed nuclei in the rare-earth region, β transitions having *log ft* < 5 are all au in character. Some *au* beta transitions are observed to have *log ft* values somewhat larger than 5 (up to 5.3, say). However, since some β transitions that are not *au* have *log ft* values >5, observation of a β transition with *log ft* does not automatically establish (or reject) it as *au*.

Observation of an au transition definitely establishes the Nilsson-orbital characters of the states involved, from which unique configuration assignments can frequently be made. For further discussion, see the related material in [6].

V. *Alpha Decay*

In α decay, the favored transition (hindrance factor (HF) < 4) connects states having the same configuration. In the odd-mass nuclides, this means that the initial and final states have the same Nilsson-orbital assignment. The α transitions to the higher-spin members of the band have increasingly larger HF's, but usually these are still smaller than those to final states associated with other Nilsson orbitals. For even-even nuclei, the favored transition connects the two ground states. Transitions to the higher-spin members of the ground-state band have increasingly larger HF's, although these may still be lower than those of transitions to states having different configurations. (Note, however, that collective quadrupole vibrations may also be populated by α transitions having comparable HF's.)

Trends for the variation of the α HF's within rotational bands have been analyzed and are given in [11].

VI. **Single-nucleon Transfer Reactions (light-ion-induced)**

In the strongly deformed nuclei, these reactions are a quite powerful tool for making $J\pi$ assignments. For populating the members of a band in odd-mass nuclei, the measured cross sections can yield the coefficients, C_{jl} ,of the (j,l) components in the wave function of the Nilsson orbital of the transferred nucleon. Since, in this case, the j value is also the J value of the level, this yields the J-value of the level. Since each Nilsson orbital contains a "unique" set of C_{jl} coefficients, the cross sections for populating the members of a band based on a Nilsson orbital form a specific pattern ("fingerprint"), from which both $J\pi$ and configuration assignments can be made with considerable confidence, providing that the spectrum is not too complex.

For transfer to even-mass nuclides, the situation is somewhat more complicated, but the cross-section patterns are still characteristic of the transferred nucleon.

An excellent review of single-nucleon-transfer reactions can be found in [12].

VII. **Coulomb Excitation**

In the deformed nuclei, the members of the ground-state rotational band are populated in Coulomb excitation with an enhanced probability relative to those of near-lying states that are not band members. If a sequence of levels having "rotational-like" energy spacings is found to be excited with enhanced probabilities (transition rates of tens of Weisskopf units or greater),this is evidence that this sequence (at least up to the first "backbend") forms the ground-state rotational band for the nuclide involved and provides reliable $J\pi$ values for the band members, assuming that at least one of the spins is known.

VIII. **The Gallagher-Moszkowski Rules**

These empirically established coupling rules [13] are useful in inferring the relative positions of the two two-quasiparticle states formed by the two different couplings of the quasiparticle constituents. In the doubly odd nuclei, the state corresponding to the parallel alignment of the projections (Σ=1/2) of the intrinsic spins (the "triplet" state) of the two odd particles should lie lower than that produced by the antiparallel (or "singlet" state) alignment. In the even-even nuclei, the opposite should be true, with the singlet state lying below the triplet.

For the odd-odd nuclei, these rules seem to have considerable experimental support (only five well established violations out of dozens of cases, see [4]). Thus, they may be used with some confidence, especially as an aid in arriving at possible ground-state configurations. For the even-even nuclides, the situation is less clear, since the excitations occur at or above the pairing gap, where the level densities are high and the coupling to the various vibrational excitations affects the singlet and triplet states differently.

A Concluding Remark

In order to see how some of these considerations can be used to extract significant nuclear-structure information in a specific case, the reader might want to read the analysis of the data on the level scheme of ^{229}Th presented in [14].

References

1. M. E. Bunker and C. W. Reich, Revs. Modern Phys. 43, 348 (1971).

2. R. R. Chasman, I. Ahmad, A. M. Friedman and J. R. Erskine, Revs. Modern Phys. 49, 833 (1977).

3. A. K. Jain, R. K. Sheline, P. C. Sood and Kiran Jain, Revs. Modern Phys. 62, 393 (1990).

4. A. K. Jain, R. K. Sheline, D. M. Headly, P. C. Sood, D. G. Burke, I. Hrivnacova, J. Kvasil, D. Nosek and R. W. Hoff, Revs. Modern Phys. 70, 843 (1998).

5. B. R. Mottelson and S. G. Nilsson, Mat.-fys. Skr. Dan. Vid. Selsk. 1, no. 8 (1959).

6. Procedures Manual for ENSDF, BNL-NCS-40503 (October 1987).

7. R. C. Greenwood et al., Nucl. Phys. A304, 327 (1978).

8. A. Bäcklin et al., Nucl. Phys. A380, 189 (1982).

9. R. G. Helmer and C. W. Reich, Nucl. Phys. A114, 649 (1968).

10. G. Alaga, K. Alder, A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Mat. Fys. Medd. 29, no. 9 (1955).

11. Y. A. Ellis and M. R. Schmorak, Nucl. Data Sheets B8, 345 (1972).

12. B. Elbek and P. O. Tjóm, Advances in Nucl. Phys. 3, 259 (1969).

13. C. J. Gallagher and S. A. Moszkowski, Phys. Rev. 111, 1282 (1958).

14. L. A. Kroger and C. W. Reich, Nucl. Phys. A259, 29 (1976).